Digital Communication Systems ECS 452

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Information-Theoretic Quantities

Office Hours:

Rangsit Library:

Tuesday 16:20-17:20

BKD3601-7:

Thursday 16:00-17:00

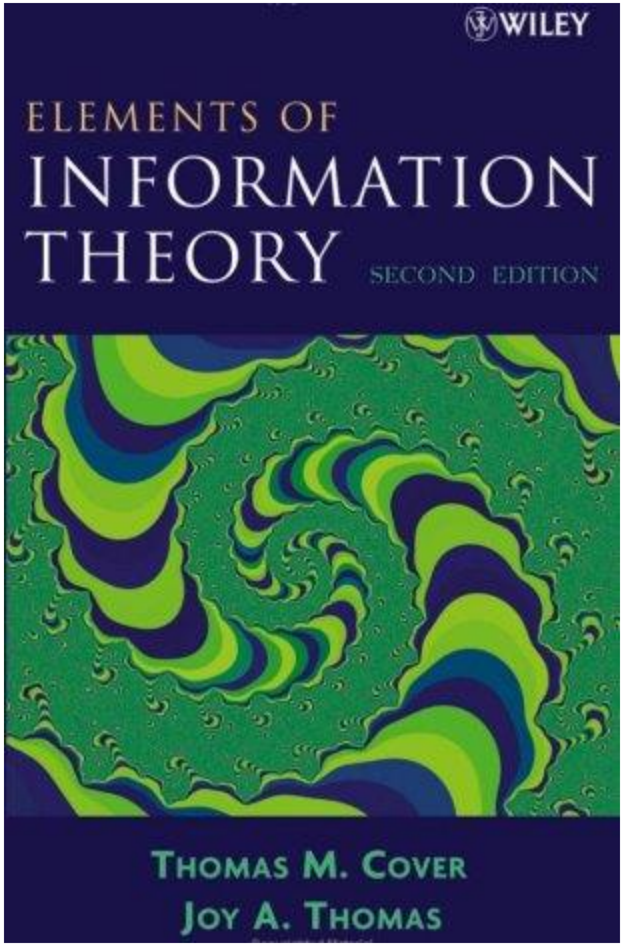
Grading System

- Coursework will be weighted as follows:

Assignments	5%
Quizzes and In-Class Exercises	10%
Class Discussion/Participation	10%
Midterm Examination • 6 Aug 2013 TIME 13:30 - 16:30	35%
Final Examination (comprehensive) •15 Oct 2013 TIME 13:30 - 16:30	40%

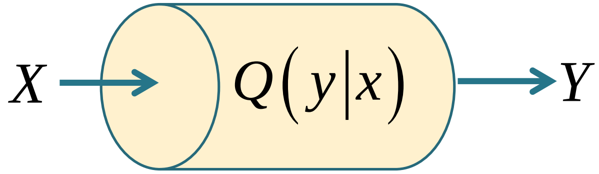
Reference for this chapter

- Elements of Information Theory
- By Thomas M. **Cover** and Joy A. **Thomas**
- 2nd Edition (Wiley)
- Chapters 2, 7, and 8
- 1st Edition available at SIIT library: Q360 C68 1991



Channel Model

- The model considered here is a simplified version of what we have seen earlier in the course.
  - In the next chapter, we will present how this model can be derived from the digital modulator-demodulator over continuous-time AWGN noise one.
- The channel input is denoted by a random variable X.
  - The pmf $p_X(x)$ is usually denoted by simply $p(x)$ and usually expressed in the form of a row vector $\underline{p}$ or $\underline{\pi}$.
  - The support S_X is often denoted by $\mathcal{X}$.
- The channel output is denoted by a random variable Y.
  - The pmf $p_Y(y)$ is usually denoted by simply $q(y)$ and usually expressed in the form of a row vector $\underline{q}$.
  - The support S_Y is often denoted by $\mathcal{Y}$.
- The channel corrupts X in such a way that when the input is $X = x$, the output Y is randomly selected from the conditional pmf $p_{Y|X}(y|x)$.
  - This conditional pmf $p_{Y|X}(y|x)$ is usually denoted by $\mathrm{Q}(y|x)$ and usually expressed in the form of a **probability transition matrix Q**.
  - $\underline{q} = \underline{p}\mathrm{Q}$



"Information" Channel Capacity

- Consider a (discrete memoryless) channel whose is $Q(y\,|\,x)$.
- The "information" channel capacity of this channel is defined as

$$C = \max_{p_X(x)} I(X;Y) = \max_{\underline{p}} I(\underline{p}, Q),$$

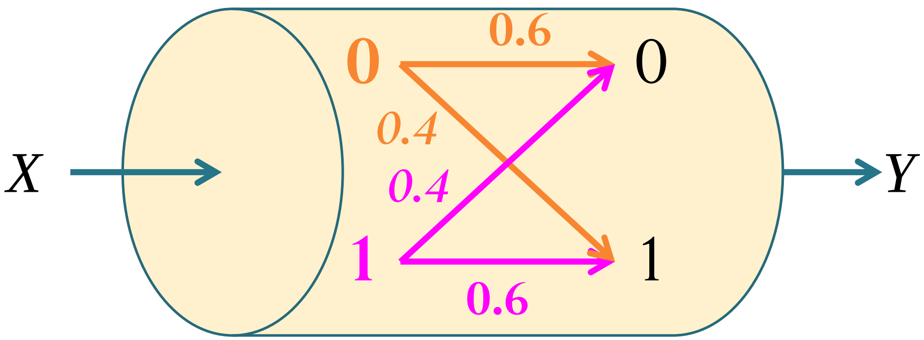
where the maximum is taken over all possible input pmf's $p_X(x)$.

- Don't confuse this Q with the Q function
- Q is given. (can not be optimize)

- Remarks:
  - In the next chapter, we shall define an "**operational**" definition of channel capacity as the highest rate in bits per channel use at which information can be sent with arbitrarily low probability of error.
  - Shannon's theorem establishes that the information channel capacity is equal to the operational channel capacity.
  - Thus, we may drop the word information in most discussions of channel capacity.

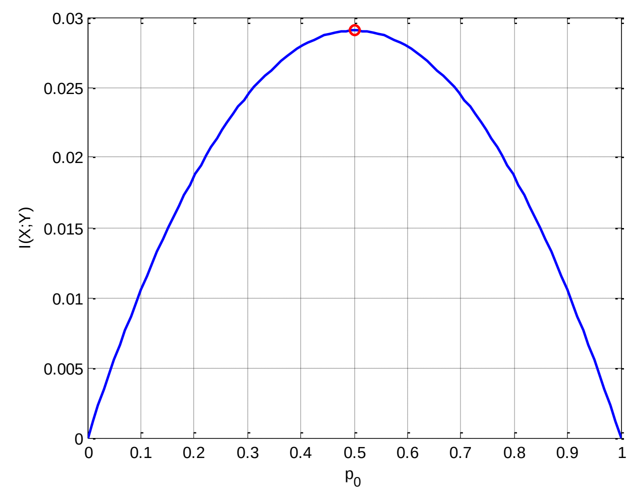
Binary Symmetric Channel (BSC)

$$H(Y|X) = H([0.4, 0.6])$$



$$\underline{q} = [p_0, 1-p_0]\begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$$

$$I(X;Y) = H(\underline{q}) - H([0.4, 0.6])$$



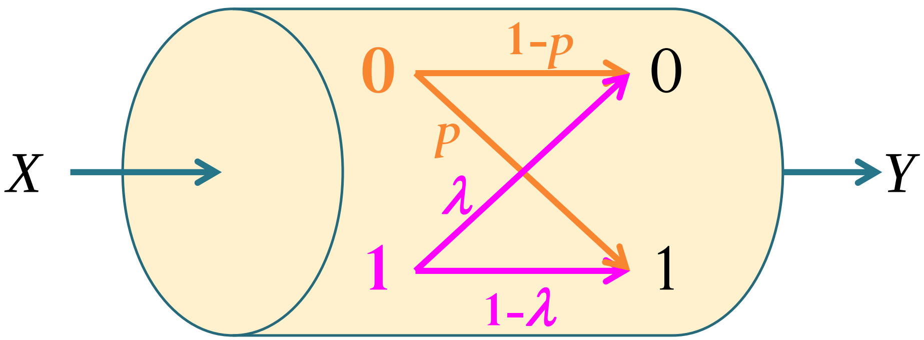
$$Q = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$$

$$\underline{p} = [p_0, 1-p_0]$$

$1 - H([0.6 \;\; 0.4])$

Capacity of 0.029 bits is achieved by $\underline{p} = [0.5, \; 0.5]$

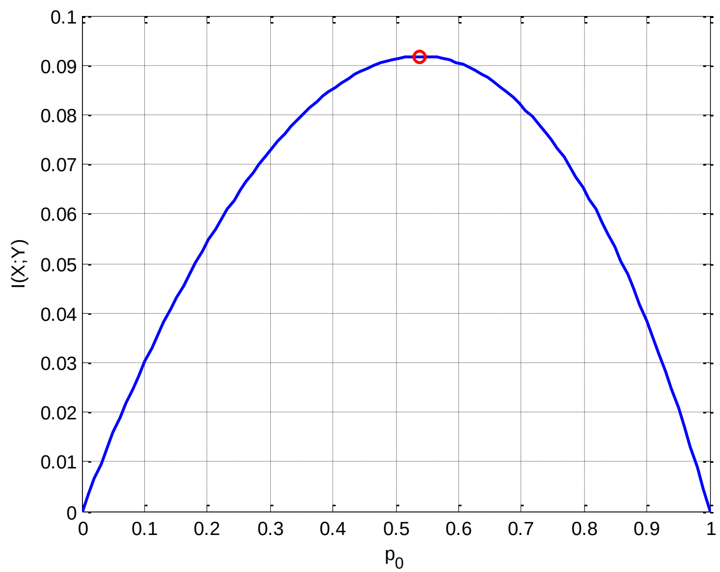
Binary Asymmetric Channel



$$Q = \begin{bmatrix} 1-p & p \\ \lambda & 1-\lambda \end{bmatrix}$$

$$p \neq \lambda$$

Ex. $p = 0.9, \quad \lambda = 0.4$



Capacity of 0.0918 bits is achieved by $\underline{p} = [0.5380, \ 0.4620]$

Iterative Calculation of C

- In general, there is no closed-form solution for capacity.
- The maximum can be found by standard nonlinear optimization techniques.
- A famous iterative algorithm, called the **Blahut–Arimoto algorithm,** was developed by Arimoto and Blahut.
  - Start with a guess input pmf $p_0(x)$.
  - For $r > 0$, construct $p_r(x)$ according to the following iterative prescription:

$$p_{r+1}(x) = \frac{p_r(x)\, c_r(x)}{\sum_x p_r(x)\, c_r(x)},$$

where

$$\log c_r(x) = \sum_y Q(y|x) \log \frac{Q(y|x)}{q_r(y)}$$

and

$$q_r(y) = \sum_x p_r(x)\, Q(y|x).$$

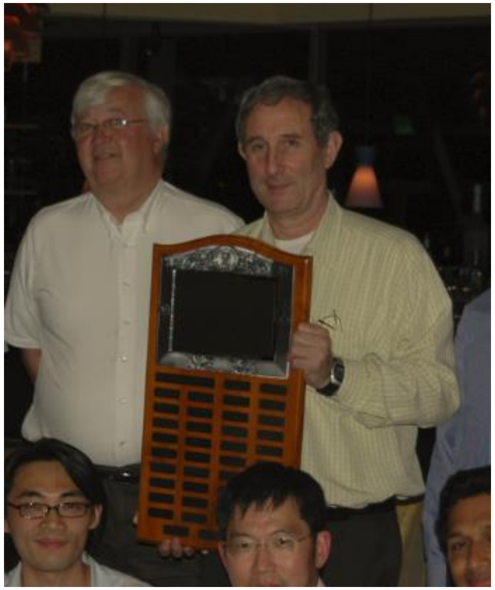
Then,

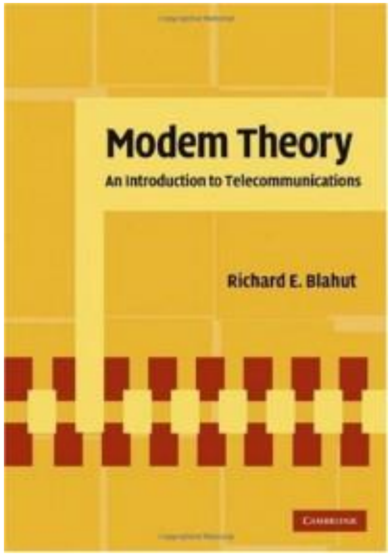
$$\log\left(\sum_x p_r(x)\, c_r(x)\right) \le C \le \log\left(\max_x c_r(x)\right).$$

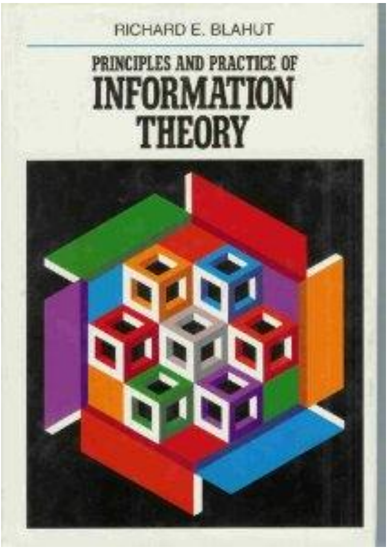
Berger plaque

Richard Blahut

- Former chair of the Electrical and Computer Engineering Department at the University of Illinois at Urbana-Champaign
- Best known for **Blahut–Arimoto algorithm (Iterative Calculation of C)**







Raymond Yeung

- BS, MEng and PhD degrees in electrical engineering from **Cornell** University in 1984, 1985, and 1988, respectively.